36. Trigonometry	• interpret and use three-figure bearings	Measured clockwise from the north, i.e. 000°–360°. e.g. Find the bearing of *A* from *B* if the bearing of *B* from *A* is 125°
	• apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle • solve trigonometrical problems in two dimensions involving angles of elevation and depression • extend sine and cosine functions to angles between 90° and 180° • solve problems using the sine and cosine rules for any triangle and the formula area of triangle $= \frac{1}{2}ab\sin C$ • solve simple trigonometrical problems in three dimensions	Angles will be quoted in, and answers required in, degrees and decimals of a degree to one decimal place. Calculations of the angle between two planes or of the angle between a straight line and plane will not be required.

Notes

https://drive.google.com/open?id=1kkHUec_9uIgMey-vZ2BATYGzrObxbKuY

Videos for understanding

https://www.youtube.com/watch?v=xE3BZXpCKqE

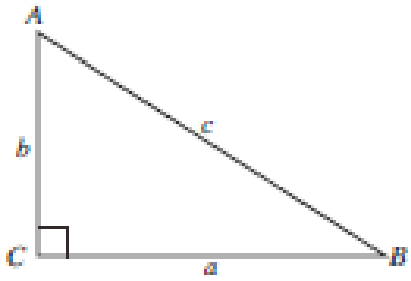
https://www.youtube.com/watch?v=jZxgmsg_82E

https://www.youtube.com/watch?v=WqBDpujbtIo

https://www.youtube.com/watch?v=I8LI7wPSvNI

UNIT 2.4 Pythagoras' Theorem and Trigonometry

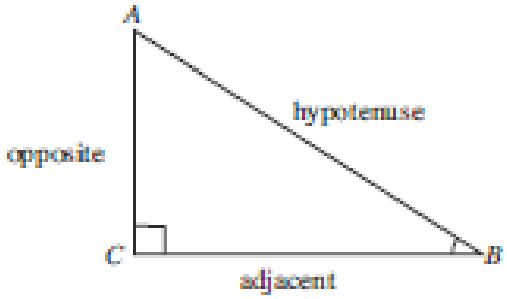
Pythagoras' Theorem



1. For a right-angled triangle ABC, if $\angle C = 90°$, then $AB^2 = BC^2 + AC^2$, i.e. $c^2 = a^2 + b^2$.

2. For a triangle ABC, if $AB^2 = BC^2 + AC^2$, then $\angle C = 90°$.

Trigonometric Ratios of Acute Angles



3. The side opposite the right angle C is called the hypotenuse. It is the longest side of a right-angled triangle.

4. In a triangle ABC, if $\angle C = 90°$,

then $\frac{AC}{AB} = \frac{\text{opp}}{\text{adj}}$ is called the sine of $\angle B$, or $\sin B = \frac{\text{opp}}{\text{hyp}}$,

$\frac{BC}{AB} = \frac{\text{adj}}{\text{hyp}}$ is called the cosine of $\angle B$, or $\cos B = \frac{\text{adj}}{\text{hyp}}$,

$\frac{AC}{BC} = \frac{\text{opp}}{\text{adj}}$ is called the tangent of $\angle B$, or $\tan B = \frac{\text{opp}}{\text{adj}}$.

Trigonometric Ratios of Obtuse Angles

5. When θ is obtuse, i.e. $90° < \theta < 180°$,

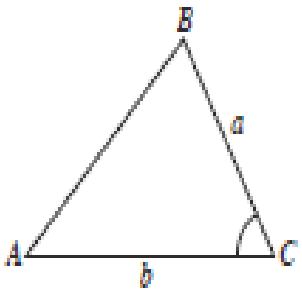
$\sin \theta = \sin (180° - \theta)$,

$\cos \theta = -\cos (180° - \theta)$.

$\tan \theta = -\tan (180° - \theta)$.

Area of a Triangle

6. Area of $\Delta ABC = \frac{1}{2} ab \sin C$



Sine Rule

7. In any ΔABC, the Sine Rule states that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

8. The Sine Rule can be used to solve a triangle if the following are given:

- two angles and the length of one side; or
- the lengths of two sides and one non-included angle.

Cosine Rule

9. In any ΔABC, the Cosine Rule states that $a^2 = b^2 + c^2 - 2bc\cos A$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

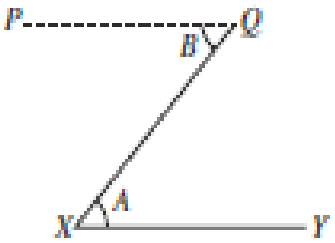
or $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

10. The Cosine Rule can be used to solve a triangle if the following are given:
    - the lengths of all three sides; or
    - the lengths of two sides and an included angle.

Angles of Elevation and Depression

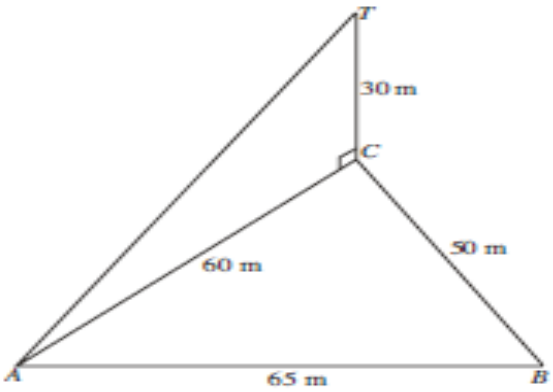


11. The angle A measured from the horizontal level XY is called the angle of elevation of Q from X.

12. The angle B measured from the horizontal level PQ is called the angle of depression of X from Q.

Example 1

In the figure, A, B and C lie on level ground such that $AB = 65$ m, $BC = 50$ m and $AC = 60$ m. T is vertically above C such that $TC = 30$ m.



Find
(i) $A\hat{C}B$,
(ii) the angle of elevation of T from A.

Solution

(i) Using cosine rule,

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos A\hat{C}B$$
$$65^2 = 60^2 + 50^2 - 2(60)(50)\cos A\hat{C}B$$
$$\cos A\hat{C}B = \frac{1875}{6000}$$
$$A\hat{C}B = 71.8° \text{ (to 1 d.p.)}$$

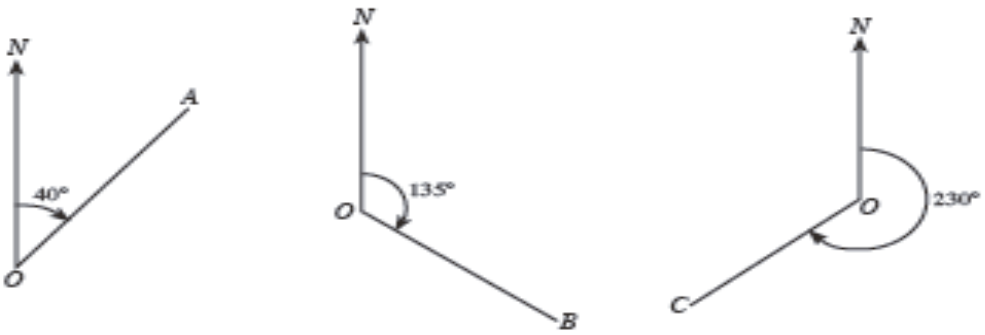
(ii) In ΔATC,

$$\tan T\hat{A}C = \frac{30}{60}$$
$$T\hat{A}C = 26.6° \text{ (to 1 d.p.)}$$

$\therefore$ Angle of elevation of T from A is 26.6°

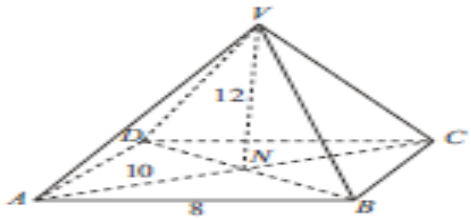
Bearings

13. The bearing of a point A from another point O is an angle measured from the north, at O, in a clockwise direction and is written as a three-digit number.
e.g.



The bearing of A from O is 040°.
The bearing of B from O is 135°.
The bearing of C from O is 230°.

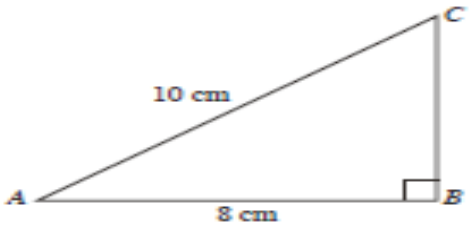
Example 3



The figure shows a pyramid with a rectangular base, $ABCD$, and vertex V. The slant edges VA, VB, VC and VD are all equal in length and the diagonals of the base intersect at N. $AB = 8$ cm, $AC = 10$ cm and $VN = 12$ cm.
(i) Find the length of BC.
(ii) Find the length of VC.
(iii) Write down the tangent of the angle between VN and VC.

Solution

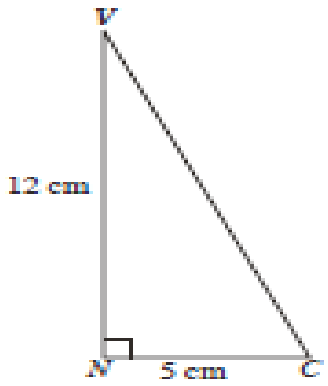
(i)



Using Pythagoras' Theorem,
$AC^2 = AB^2 + BC^2$
$10^2 = 8^2 + BC^2$
$BC^2 = 36$
$BC = 6$ cm

(ii) $CN = \frac{1}{2}AC$

$= 5$ cm



Using Pythagoras' Theorem,

$VC^2 = VN^2 + CN^2$

$= 12^2 + 5^2$

$= 169$

$VC = 13$ cm

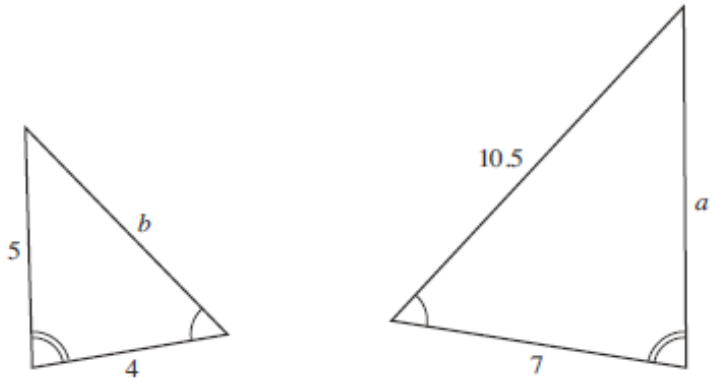
(iii) The angle between VN and VC is $C\hat{V}N$.

In ΔVNC,

$\tan C\hat{V}N = \frac{CN}{VN}$

$= \frac{5}{12}$

1 The two triangles below are similar.
The lengths are in centimetres. SP18/01/13



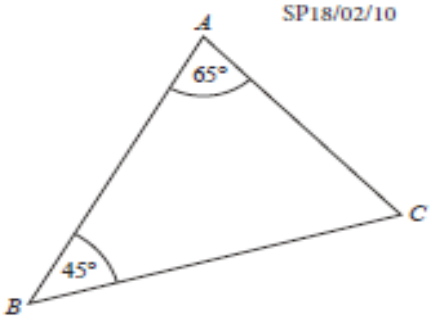
Calculate a and b.

Answer $a =$ .....

$b =$ ..... [3]

2 (a)

SP18/02/10



In triangle ABC, $A\hat{B}C = 45°$ and $B\hat{A}C = 65°$.
AC is 5 cm shorter than BC.

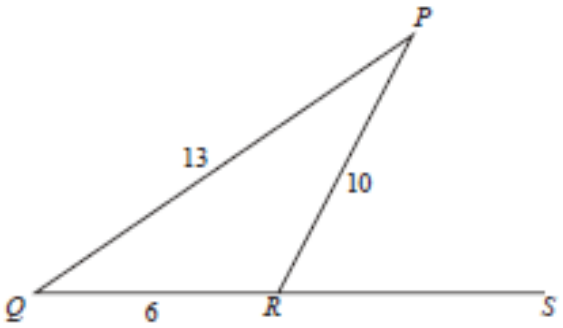
(i) Show that $BC = \dfrac{5 \sin 65}{\sin 65 - \sin 45}$.

[3]

(ii) Find the length of BC.

Answer $BC =$ ..... cm [1]

(b)



In triangle PQR, $PQ = 13$ cm, $QR = 6$ cm and $RP = 10$ cm.
QR is produced to S.

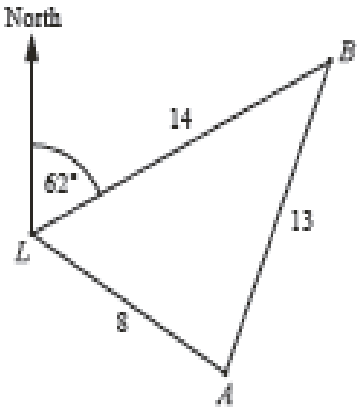
(i) Find the value of cos $P\hat{R}Q$, giving your answer as a fraction in its simplest form.

Answer ..... [3]

(ii) Hence write down the value of cos $P\hat{R}S$.

Answer ..... [1]

3



M/J19/21/11

NOT TO SCALE

The diagram shows the positions of two ports, A and B, and a lighthouse L.
The bearing of B from L is 062°.
$AB = 13$ km, $BL = 14$ km and $AL = 8$ km.

(a) Calculate the bearing of A from L.

..... [4]

(b) A boat is located at C.
C is 11 km from B and $B\hat{C}A = 90°$.
The boat travels to port A in a straight line.

Find the distance the boat travels.

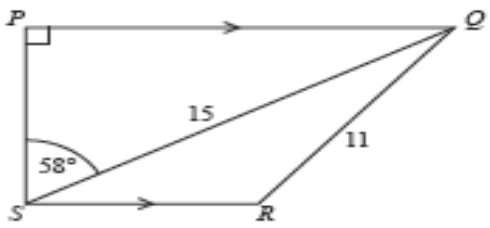
..... km [2]

(c) The boat then travels in a straight line from port A to port B.
It travels at an average speed of 3.75 km/h.

Calculate the time taken for the boat to travel from port A to port B.
Give your answer in hours and minutes.

..... hours ..... minutes [2]

(4) M/J19/21/5(b)



NOT TO SCALE

$PQRS$ is a trapezium with PQ parallel to SR and $S\hat{P}Q = 90°$.
$SQ = 15$ cm, $QR = 11$ cm and $PS\hat{Q} = 58°$.

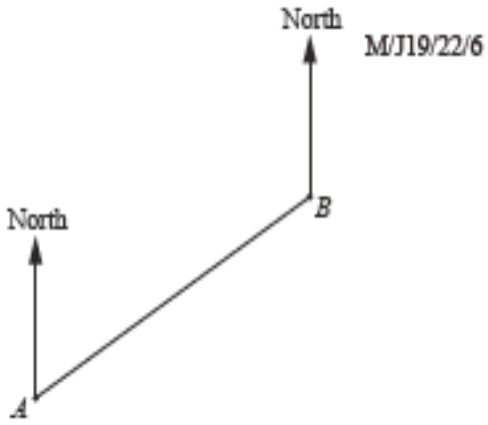
(i) Calculate PS.

$PS =$ ..... cm [2]

(ii) Calculate the obtuse angle SRQ.

Angle $SRQ =$ ..... [4]

5



The diagram shows the positions of two boats, A and B, drawn to a scale of $1 : m$.
The actual distance between the two boats is 4 km.

(a) Find m, giving your answer correct to 1 significant figure.

$m =$ ..... [2]

(b) Measure the bearing of A from B.

..... [1]

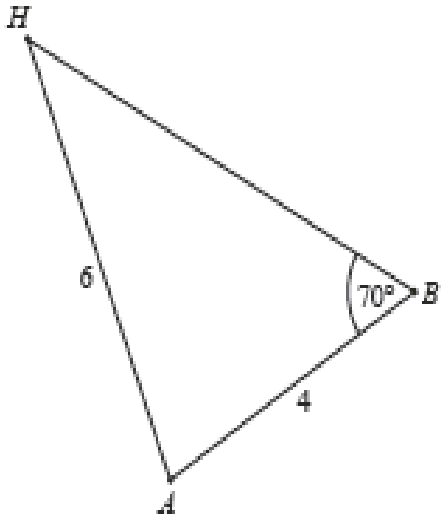
(c) A third boat is positioned at C.
C is on a bearing of 120° from A and on a bearing of 195° from B.

Find and label C on the diagram. [2]

(d) Find, by measurement, the actual distance in kilometres from A to C.

..... km [2]

(e)



NOT TO SCALE

The diagram shows the positions of the boats, A and B, and a harbour, H.
$AB = 4$km, $AH = 6$km and $A\hat{B}H = 70°$.

(i) Calculate $A\hat{H}B$.

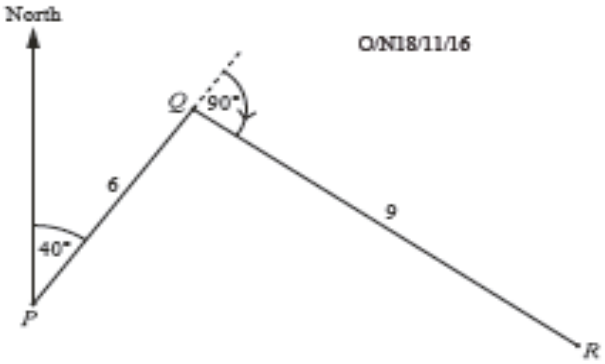
$A\hat{H}B$ = ... [3]

(ii) The boat at A travels in a straight line to the harbour at H.
The average speed of the boat is p km/h.
It takes 12 minutes 20 seconds for the boat to travel from A to H.

Calculate p.

p = ... [3]

6



A boat travels from P to Q.
At Q, it turns through 90° and travels to R as shown in the diagram.

It then returns from R to Q, and then to P, following the same route in reverse.
$PQ = 6$ km and $QR = 9$ km.

The first part of the journey, from P to Q to R, takes 3 hours.
The return part of the journey, from R to Q to P, takes 2 hours.

(a) Calculate the average speed for the whole journey from P to Q to R and back from R to Q to P.

Answer ..... km/h [2]

(b) The bearing of Q from P is 040°.

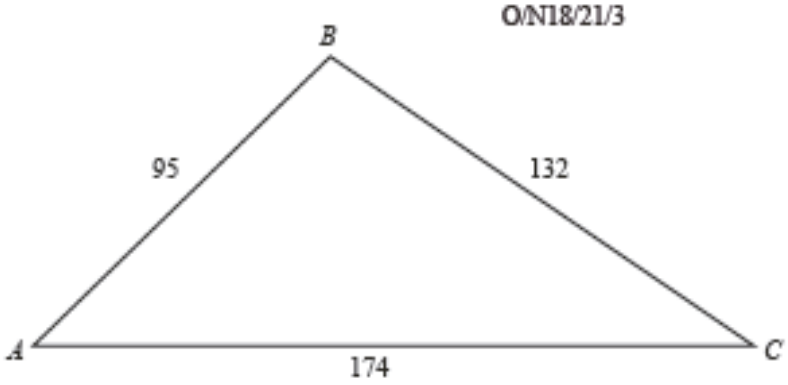
(i) Calculate the bearing of R from Q.

Answer ..... [1]

(ii) Calculate the bearing of P from Q.

Answer ..... [1]

7 O/N18/21/3



The diagram shows a triangular field ABC.
$AB = 95$ m, $BC = 132$ m and $AC = 174$ m.

(a) Show that $B\hat{A}C = 48.6°$, correct to 1 decimal place.

[3]

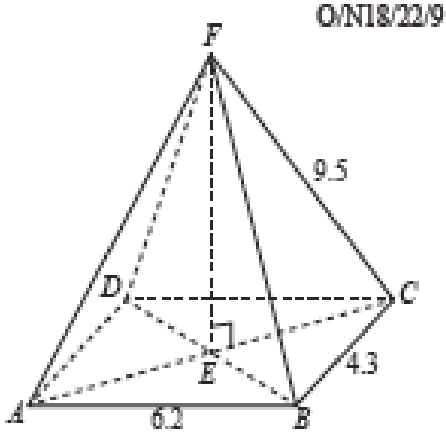
(b) The field is sown with flower seeds.
Each square metre of the field is sown with 3 grams of seed.
The seed costs \$8.50 for 100 grams.

Calculate the cost of the flower seed needed for the field.

Answer \$ ..... [4]

8 [Volume of a pyramid = $\frac{1}{3}$ × base area × height]

O/N18/22/9



The diagram shows a pyramid with a rectangular, horizontal base.
Vertex F of the pyramid is vertically above the centre of the base, E.
$AB = 6.2$ cm and $BC = 4.3$ cm.
The length of each sloping edge of the pyramid is 9.5 cm.

(a) Show that the height, EF, of the pyramid is 8.72 cm, correct to 3 significant figures.

[4]

(b) Calculate the volume of the pyramid.

Answer ..... cm^3 [2]

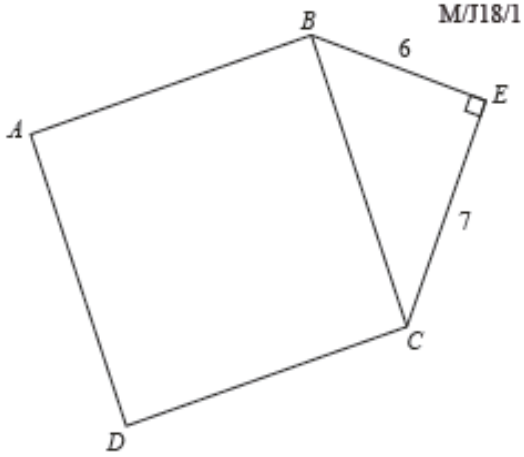
(c) Calculate angle AFB.

Answer ..... [3]

(d) Calculate the angle of elevation of F from the midpoint of AB.

Answer ..... [2]

9 M/J18/11/17

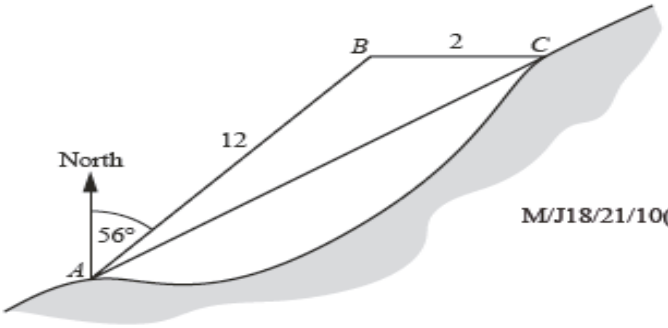


The diagram shows a square $ABCD$ joined to a right-angled triangle BEC.
$BE = 6$ cm and $EC = 7$ cm.

Calculate the area of the pentagon, $ABECD$.

Answer ..... cm^2 [3]

10



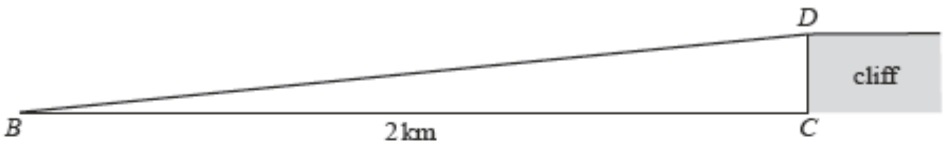
M/J18/21/10(b)

The bearing of B from A is 056°.
B is 2 km due west of C.

Calculate AC.

Answer ..... km [4]

(c)

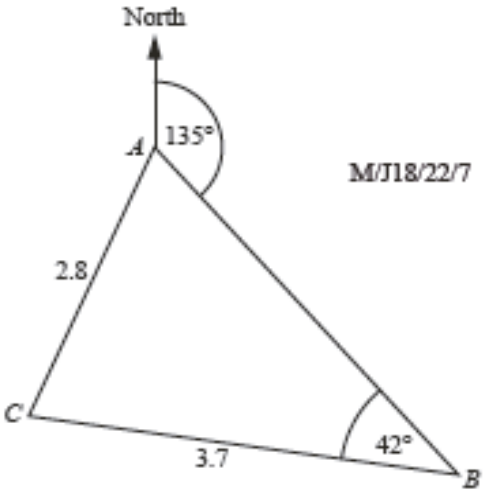


C is the base of a cliff.
The top of the cliff, D, is vertically above C.
DC is perpendicular to BC and $DC = 105$ m.

Calculate the angle of elevation of D from B.

Answer ..... [2]

11



M/J18/22/7

A yacht sails the triangular route shown.
The bearing of B from A is 135°.
$BC = 3.7$km, $AC = 2.8$km and $A\hat{B}C = 42°$.

(a) Show that $C\hat{A}B = 62.2°$, correct to 1 decimal place.

[3]

(b) Find the bearing of A from C.

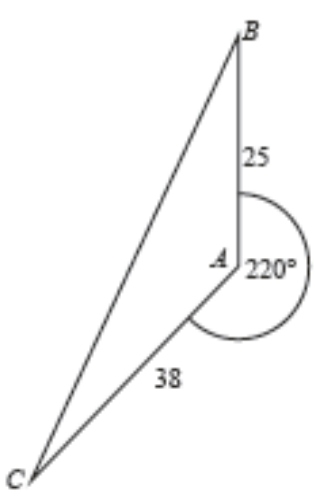
Answer ..... [2]

(c) The yacht sails from A to B to C to A.

Calculate the total length of the route.

Answer ..... km [4]

12



O/N17/21/3

The diagram shows the positions of three towns, A, B and C.
B is due north of A and the bearing of C from A is 220°.
$AB = 25$ km and $AC = 38$ km.

(a) Find the bearing of A from C.

Answer ..... [1]

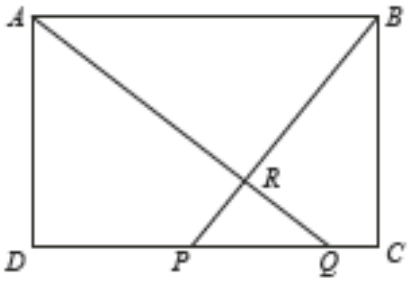
(b) Show that $BC = 59.4$ km correct to 3 significant figures.

[3]

(c) Calculate the bearing of C from B.

Answer ..... [4]

13



O/N17/21/11

$ABCD$ is a rectangle.
P and Q are points on DC.
AQ and BP intersect at R.

(a) Prove that triangle ARB is similar to triangle QRP.
Give a reason for each statement you make.

.....

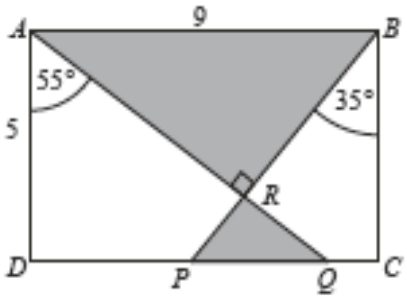
.....

.....

.....

..... [3]

(b)



In rectangle $ABCD$, $AB = 9$ cm and $AD = 5$ cm.
$D\hat{A}Q = 55°$, $C\hat{B}P = 35°$ and AQ is perpendicular to BP.

(i) Calculate AQ.

Answer ..... cm [2]

(ii) Calculate AR.

Answer ..... cm [2]

(iii) Calculate the area of triangle ARB.

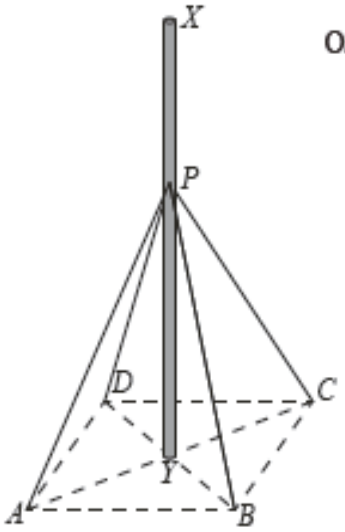
Answer ..... cm^2 [2]

(iv) Calculate the total area shaded in the rectangle.

Answer ..... cm^2 [3]

14

O/N17/22/11



A vertical mast, XY, is positioned on horizontal ground.
The mast is supported by four cables attached to the mast at P and to the ground at points A, B, C and D.
Y is the centre of the square $ABCD$.
$PY = 7.50$ m.

(a) Given that $AB = 3.65$ m, show that $AY = 2.58$ m correct to 3 significant figures.

[3]

(b) Calculate the length of one of the cables used to support the mast.

Answer ..... m [2]

(c) Calculate $A\hat{P}B$.

Answer ..... [3]

(d) The angle of elevation of X from A is 77.0°.

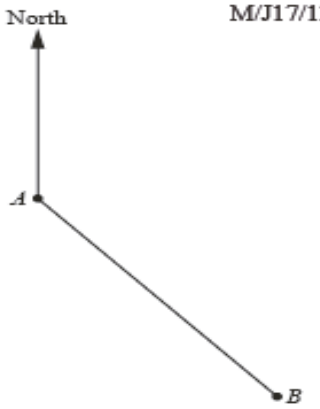
(i) Calculate the height, XY of the mast.

Answer ..... m [2]

(ii) Calculate the angle of elevation of X from the midpoint of AB.

Answer ..... [2]

15 The diagram shows the position of two villages A and B. M/J17/12/5



(a) Measure the bearing of B from A.

Answer ..... [1]

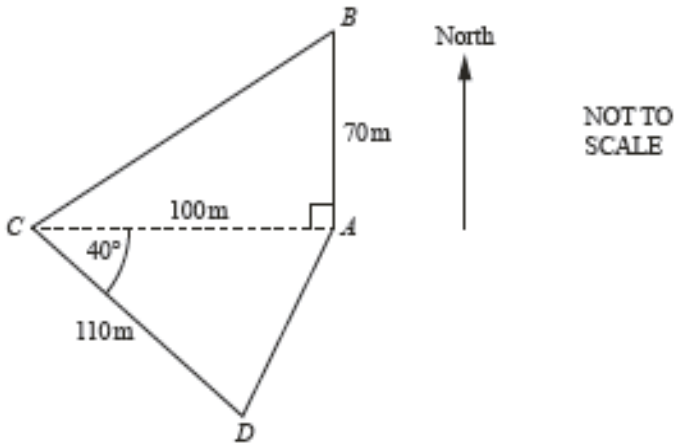
(b) The bearing of village C from A is 265°.

Work out the bearing of A from C.

Answer ..... [1]

(IGCSE QUESTION)

16



The diagram shows a field *ABCD*.

(a) Calculate the area of the field *ABCD*.

.....m^2 [3]

(b) Calculate the perimeter of the field *ABCD*.

..... m [5]

(c) Calculate the shortest distance from A to CD.

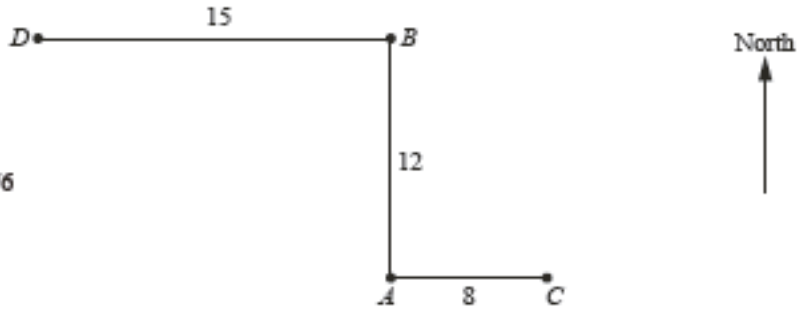
..... m [2]

(d) B is due north of A.

Find the bearing of C from B.

..... [3]

17 (a)



M/J17/21/6

A, B, C and D are four towns.
B is 12 km due north of A, C is 8 km due east of A and D is 15 km due west of B.

(i) Calculate the distance of B from C.

Answer ..... km [2]

(ii) Calculate the bearing of A from D.

Answer ..... [3]

(b)

The diagram shows the position of a clock tower, T, and a statue, S, drawn to a scale of 1 cm to 75 m.

(i) Using measurements taken from the diagram, find the actual distance between T and S.

Answer ..... m [2]

(ii) A fountain, F, is situated 450 m from T on a bearing of 210°.

Draw and label F. [2]

(iii) Using measurements taken from the diagram, find the bearing of F from S.

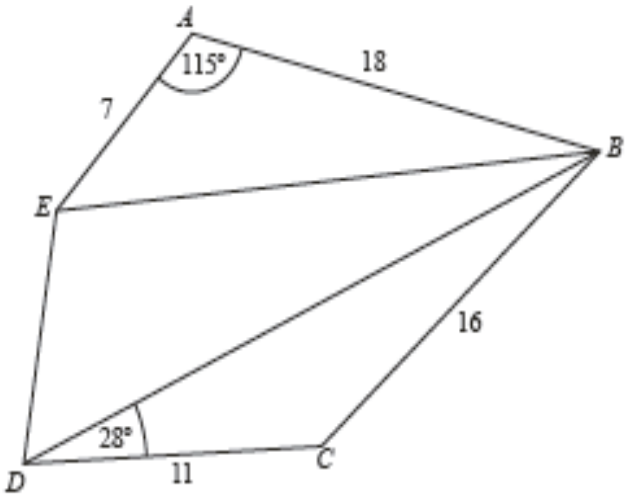
Answer ..... [1]

18 (a) Calculate the interior angle of a regular nine-sided polygon.

M/J17/21/9

Answer ..... [2]

(b)



ABCDE is a pentagon.
$AB = 18$ cm, $BC = 16$ cm, $CD = 11$ cm and $EA = 7$ cm.
$E\hat{A}B = 115°$ and $B\hat{D}C = 28°$.

(i) Show that $BE = 21.9$ cm, correct to 3 significant figures.

[3]

(ii) Calculate angle DBC.

Answer ..... [3]

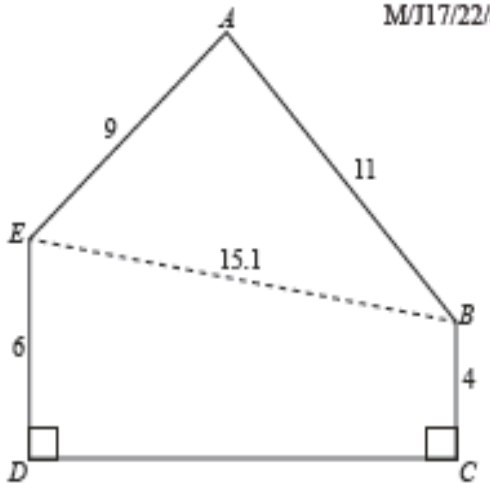
(iii) The perimeter of the pentagon is 62 cm.

Given that the area of triangle BDE is 109 cm^2, calculate the **obtuse** angle DEB.

Answer ..... [4]

19

M/J17/22/8



ABCDE is the cross-section of a building.
All the lengths are given in metres.

(a) Calculate *DC*.

Answer ..... m [3]

(b) Calculate angle *EAB*.

Answer ..... [3]

(c) Calculate the area of the cross-section.

Answer ..... m^2 [4]

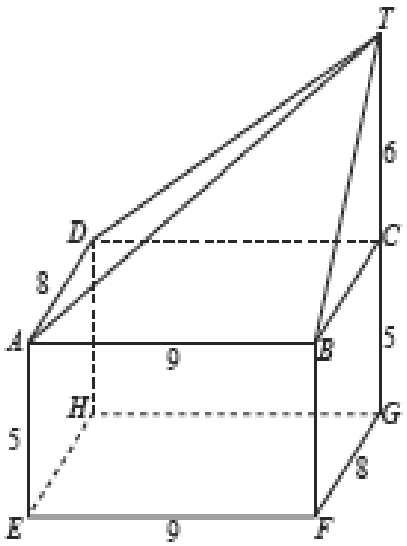
(d) A model of the building is made using the scale 1 : 50.

What is the area of the cross-section of the model?
Give your answer in square centimetres.

Answer ..... cm^2 [2]

20

O/N16/21/6



The four walls of a building are faces of a cuboid $ABCDEFGH$.
T is vertically above C and G, so $A\hat{B}T = A\hat{D}T = 90°$.

The cuboid has length 9 m, width 8 m and height 5 m.
$TC = 6$ m.

(a) Calculate the length of DT.

Answer ..... m [2]

(b) The roof is formed by four triangles, ABT, BCT, CDT and DAT.

Calculate the total surface area of the roof.

Answer ..... m^2 [3]

(c) **[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height]**

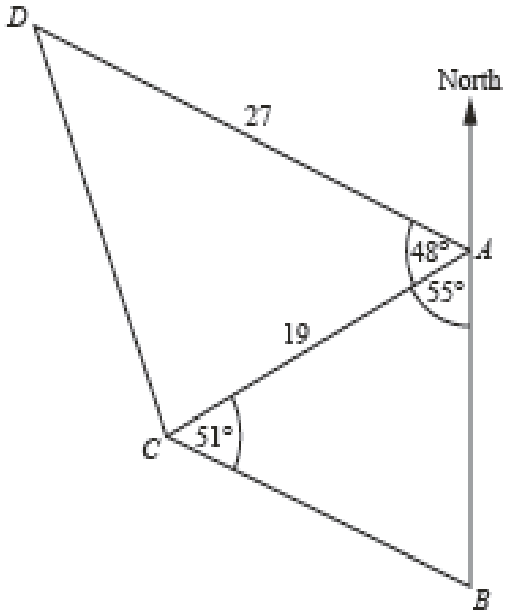
Calculate the **total** volume of the building.

Answer ..... m^3 [2]

(d) Calculate the angle of elevation of T from H.

Answer ..... [3]

21 O/N16/21/7



The diagram shows the positions of four islands at A, B, C and D.
A is due north of B.
$D\hat{A}C = 48°$, $C\hat{A}B = 55°$ and $B\hat{C}A = 51°$.
$AC = 19$ km and $AD = 27$ km.

(a) Calculate the bearing of D from A.

Answer ..... [1]

(b) Calculate the bearing of A from C.

Answer ..... [1]

(c) Calculate the distance between A and B.

Answer ..... km [3]

(d) Calculate the distance between D and C.

Answer ..... km [3]

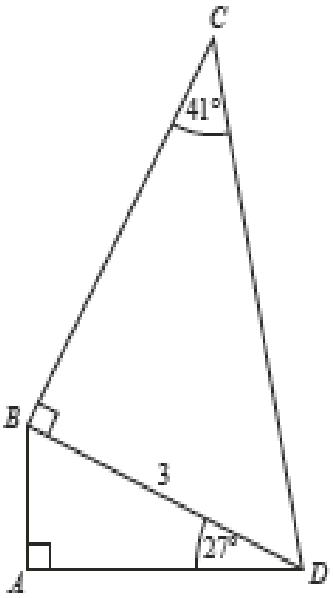
(e) A boat leaves D and sails, at a constant speed, in a straight line to A.
It takes 3 hours and 36 minutes to sail from D to A.
X is the point on DA that is closest to C.

Calculate the time, correct to the nearest minute, the boat takes to travel from D to X.

Answer ..... [4]

22 (a)

O/N16/22/4



In the framework $ABCD$, $BD = 3$ m.
$B\hat{D}A = 27°$, $B\hat{C}D = 41°$. $D\hat{B}C$ and $D\hat{A}B$ are right angles.

(i) Find AD.

Answer ..... m [2]

(ii) Find CD.

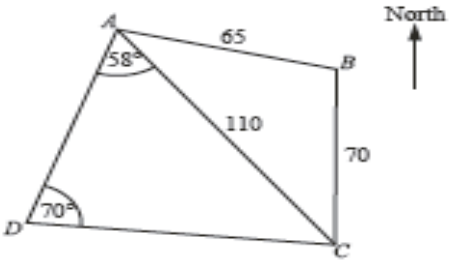
Answer ..... m [3]

(b) In triangle PQR, $PQ = 3\,\text{m}$ and $QR = 5\,\text{m}$.
The area of triangle $PQR = 6\,\text{m}^2$.

Find the two possible values of $P\hat{Q}R$.

Answer $P\hat{Q}R =$ ..... or ..... [3]

23



$ABCD$ is a level playing field.
$AB = 65\,\text{m}$, $BC = 70\,\text{m}$ and $CA = 110\,\text{m}$.
$C\hat{D}A = 70°$, $D\hat{A}C = 58°$ and C is due South of B.

O/N16/22/9

(a) Calculate the bearing of A from C.

Answer ..... [4]

(b) Calculate AD.

Answer ..... m [3]

(c) There are two vertical trees, AX and CY, each of height 17 m, one at each end of the path AC.

(i) Calculate the angle of elevation of Y from B.

Answer ..... [2]

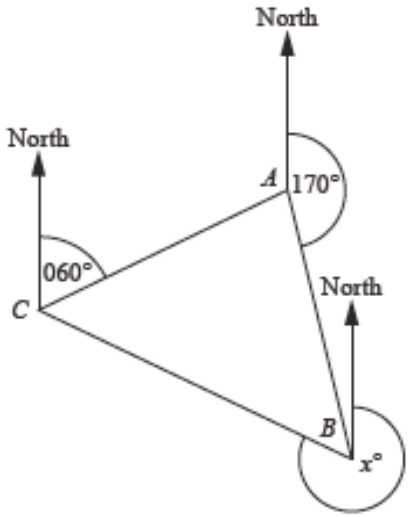
(ii) A bird flies in a straight line from X to Y.
It takes 24 seconds.

Calculate the average speed of the bird.
Give your answer in kilometres per hour.

Answer ..... km/h [3]

24 In the diagram, the bearing of B from A is 170°.
The bearing of A from C is 060°.
The bearing of C from B is x°.

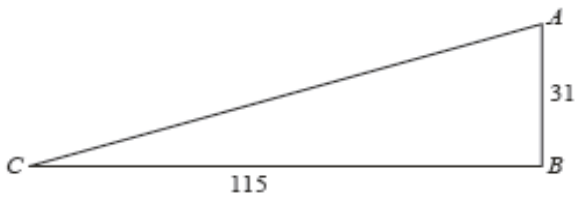
M/J16/12/17



Given that triangle ABC is isosceles, find the three possible values of x.

Answer $x =$ ..... or ..... or ..... [3]

25 **(a)** M/J16/21/5

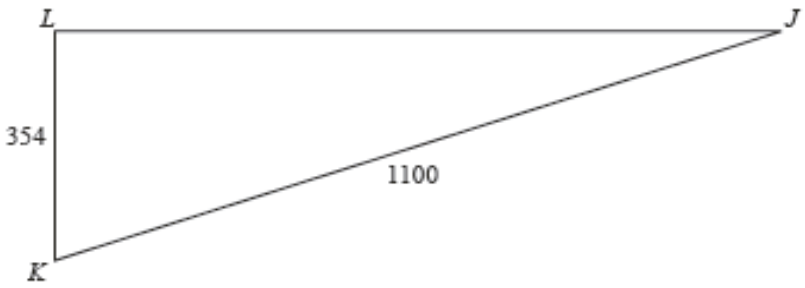


AB is vertical and CB is horizontal.
$AB = 31$ m and $CB = 115$ m.

Calculate the angle of depression of C from A.

Answer ..... [3]

(b)



J and K are two positions at sea.
The base of a lighthouse is at L.
J is due East of L and K is due South of L.
$KL = 354$ m and $KJ = 1100$ m.

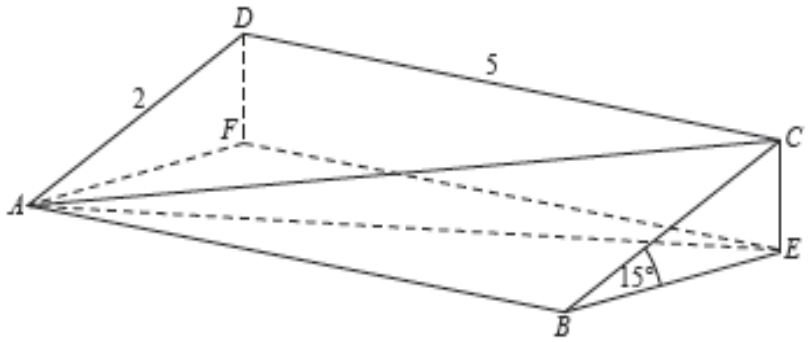
(i) Calculate $L\hat{J}K$.

Answer ..... [2]

(ii) Hence find the bearing of K from J.

Answer ..... [1]

26 **(a)**



ABCD represents the rectangular sloping surface of a triangular prism.
ABEF is a horizontal rectangle. *CE* and *DF* are vertical.
$C\hat{B}E = 15°$, $DC = 5$ m and $AD = 2$ m.

(i) Calculate *AC*.

M/J16/21/9

Answer ..... m [2]

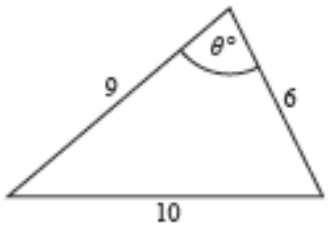
(ii) Calculate *CE*.

Answer ..... m [2]

(iii) Calculate $F\hat{A}E$.

Answer ..... [4]

(b) (i)



A triangle has sides of 10 cm, 9 cm and 6 cm, and an angle of $\theta°$, as shown in the diagram.

Calculate θ.

Answer ..... [3]

(ii) The triangle KGH has sides of a cm, b cm and c cm as shown in the diagram.

It is given that $K\hat{G}H$ is an obtuse angle.

Complete the statement below using **one** of the symbols $< \leqslant = \geqslant >$.

c^2 ..... $(a^2 + b^2)$ [1]

Marking Scheme

1	(a =) 8.75 **oe** (b =) 6 **oe**	**3**	**B2** for one correct **www** or **B1** for $\frac{4}{7}$ or $\frac{7}{4}$ **oe** seen

2(a)(i)	$\frac{5\sin 65}{\sin 65 - \sin 45}$ correctly obtained	**3**	**M1** for $\frac{BC}{\sin 65} = \frac{AC}{\sin 45}$ **oe** **soi** and **B1** for $AC = BC - 5$ **oe**
2(a)(ii)	22.7 to 22.75	**1**	
2(b)(i)	$-\frac{11}{40}$ **isw**	**3**	**M2** for $13^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos PRQ$ Or **M1** for $13^2 = 6^2 + 10^2 \pm (2) \times 6 \times 10 \times \cos PRQ$ **A1** for $\frac{33}{120}$ or for $-\frac{33}{60}$
2(b)(ii)	$\frac{11}{40}$ **ft**	**1**	

3(a)	128[.0]° or 128.03° to 128.04°	**4**	**B3** for 66[.0] or 66.03 to 66.04 **or** **M2** for [cos L] = $\frac{14^2 + 8^2 - 13^2}{2 \times 14 \times 8}$ **or** **M1** for $13^2 = 14^2 + 8^2 - 2 \times 14 \times 8 \times \cos L$ After 0 scored, **SC1** for 62 + *their* $B\hat{L}A$
3(b)	6.93 or 6.92	**2**	**M1** for $13^2 - 11^2$ isw
3(c)	3 hours 28 minutes	**2**	**M1** for $\frac{13}{3.75}$

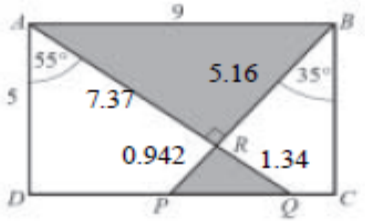
4(i)	7.95 or 7.948 to 7.949	**2**	**M1** for cos 58 = $\frac{PS}{15}$ oe
4(ii)	133.7 or 133.72 to 133.73	**4**	**B1** for 32 **and** **M2** for sin $R = \frac{15 \times \sin \textit{their} 32}{11}$ **or** **M1** for $\frac{15}{\sin R} = \frac{11}{\sin \textit{their} 32}$ oe

5(a)	80 000	**2**	**B1** for answer figs 8 or 400 000 seen or figs5 : figs4 seen
5(b)	235 to 240	**1**	
5(c)	Correct position of C	**2**	**B1** for bearing of 120° from A or 195° from B
5(d)	2.56 to 2.96	**2**	Dep on 2 marks in (c) **B1FT** for correct measurement of *their* AC
5(e)(i)	38.8 or 38.78 to 38.79	**3**	**M2** for $\sin[\ldots]=\frac{4\sin 70}{6}$ or $\sin^{-1}\left(\frac{4\sin 70}{6}\right)$ or **M1** for $\frac{4}{\sin[\ldots]}=\frac{6}{\sin 70}$ oe
5(e)(ii)	29.2 or 29.18 to 29.19	**3**	**M2** for $\frac{6}{12\frac{1}{3}}\times 60$ oe or **M1** for $\frac{6}{\textit{their}\ \text{time}}$ or $\frac{6000}{\textit{their}\ \text{time}}$
6(a)	6 nfww	**2**	**M1** for (2 × (6 + 9)) / (time in hours)
6(b)(i)	130°	**1**	
6(b)(ii)	220°	**1**	
7(a)	$\cos A=\frac{95^2+174^2-132^2}{2\times 95\times 174}$	**M2**	or **M1** for $132^2=95^2+174^2-2\times 95\times 174\times\cos A$
	$A = 48.56[7\ldots]$ or 48.57	**A1**	
7(b)	1580 to 1581	**4**	**M1** for $\frac{1}{2}\times 95\times 174\times\sin 48.6$ AND **M2** for *their* area × 3 ÷ 100 × 8.50 or **M1** for two operations correct in *their* area × 3 ÷ 100 × 8.50 or for 3 ÷ 100 × 8.50 **soi**

8(a)	$AE^2 = \left(\frac{4.3}{2}\right)^2 + \left(\frac{6.2}{2}\right)^2$ oe or $FX^2 = 9.5^2 - \left(\frac{4.3}{2}\right)^2$ or $FY^2 = 9.5^2 - \left(\frac{6.2}{2}\right)^2$	**M2**	**M1** for $AC^2 = 4.3^2 + 6.2^2$ oe or $FX^2 + \left(\frac{4.3}{2}\right)^2 = 9.5^2$ or $FY^2 + \left(\frac{6.2}{2}\right)^2 = 9.5^2$
	$[EF^2 =]\ 9.5^2 - \text{their } AE^2$ oe or $[EF^2 =]\ \text{their} FX^2 - \left(\frac{6.2}{2}\right)^2$ or $[EF^2 =]\ \text{their} FY^2 - \left(\frac{4.3}{2}\right)^2$	**M1**	Dep on M2
	8.718 to 8.719	**A1**	
8(b)	77.47 to 77.50	**2**	**M1** for $\frac{1}{3} \times 6.2 \times 4.3 \times 8.72$
8(c)	38.1° or 38.09…°	**3**	**M2** for $2\sin^{-1}\left(\frac{3.1}{9.5}\right)$ oe or **M1** for $\sin^{-1}\left(\frac{3.1}{9.5}\right)$ oe Alternative method: **M2** for $\cos AFB = \frac{9.5^2 + 9.5^2 - 6.2^2}{2 \times 9.5 \times 9.5}$ or **M1** for $6.2^2 = 9.5^2 + 9.5^2 - 2 \times 9.5 \times 9.5 \times \cos AFB$
8(d)	76.1° or 76.2° or 76.14 to 76.18°	**2**	**M1** for $\tan[\ldots] = \frac{8.72}{4.3 \div 2}$ oe

9	106	**3**	**M1** for [BC² =] 6² + 7² or better and **M1** for [area triangle BCE =] $\frac{6 \times 7}{2}$ or 21
10(b)	13.7 or 13.70…	**4**	**B1** for 146° AND **M2** for $\sqrt{12^2 + 2^2 - 2 \times 12 \times 2 \times \cos 146}$ or **M1** for $12^2 + 2^2 - 2 \times 12 \times 2 \times \cos 146$ *Alternative* **B1** for 9.95 or 9.948 to 9.949 or 6.71[0...] AND **M2** for $\sqrt{\text{their}6.71^2 + (\text{their}9.94 + 2)^2}$ or **M1** for *their* $6.71^2 + (\text{their}9.94 + 2)^2$
10(c)	3.0 or 3.00 to 3.01	**2**	**M1** for tan .. = $\frac{\text{figs}105}{\text{figs}2}$ oe

11(a)	$\sin CAB = \frac{3.7\sin 42}{2.8}$ OR $C\hat{A}B = \sin^{-1}\left(\frac{3.7\sin 42}{2.8}\right)$ OR $\frac{\sin CAB}{3.7} = \frac{\sin 42}{2.8}$ **and** sin=0.88[42...]	**M2**	**M1** for $\frac{3.7}{\sin CAB} = \frac{2.8}{\sin 42}$ oe
	$C\hat{A}B = 62.15[4\ldots]$	**A1**	
11(b)	[0]17.2°	**2**	**M1** for 135 + 62.2 – 180 oe
11(c)	10.5 to 10.6	**4**	**B3** for 4.05 to 4.06 OR **M2** for $\sqrt{2.8^2 + 3.7^2 - 2\times 2.8\times 3.7\times\cos(180-42-62.2)}$ oe or **M1** for $2.8^2 + 3.7^2 - 2\times 2.8\times 3.7\times\cos(180-42-62.2)$ oe OR **M2** for $\frac{2.8\sin(180-42-62.2)}{\sin 42}$ oe or **M1** for $\frac{\sin(180-42-62.2)}{AB} = \frac{\sin 42}{2.8}$ oe OR **M2** for $\frac{3.7\sin(180-42-62.2)}{\sin 62.2}$ oe or **M1** for $\frac{\sin(180-42-62.2)}{AB} = \frac{\sin 62.2}{3.7}$ oe OR **B1** for $A\hat{C}B = 75.8$
12(a)	040	**1**	
12(b)	$BC = \sqrt{25^2 + 38^2 - 2\times 25\times 38\cos(360-220)}$	**M2**	or **M1** for $25^2 + 38^2 - 2\times 25\times 38\times\cos(360-220)$
	BC = 59.36 to 59.37	**A1**	

12(c)	204.1 to 204.3[2…]	4	**B3** for 24.1 to 24.3[2…] OR **M2** for $\sin B = \frac{38 \times \sin(360-220)}{59.4}$ or **M1** for $\frac{\sin B}{38} = \frac{\sin(360-220)}{59.4}$ and **M1** for 180 + *their B*
13(a)	$\angle ARB = \angle PRQ$, [vertically] opposite $\angle RAB = \angle RQP$, alternate [angles] $\angle RBA = \angle RPQ$ alternate [angles] ΔARB and ΔQRP similar, equal angles	3	**B1** for one pair of angles stated with reason or for two pairs with no reasons or incorrect reasons **B1** for a further correct pair of angles with reason
13(b)(i)	[AQ =] 8.72 or 8.717[…]	2	**M1** for $\cos 55 = \frac{5}{AQ}$ or $\sin 35 = \frac{5}{AQ}$ oe
13(b)(ii)	[AR =] 7.37[2…]	2	**M1** for $\cos 35 = \frac{AR}{9}$ or $\sin 55 = \frac{AR}{9}$ oe
13(b)(iii)	[Area ARB =] 18.8 to 19.2[…] or **FT** *their AR*	2	**M1** for $\frac{1}{2} \times$ *their* $7.37 \times 9 \times \sin 35$ oe Or $\frac{1}{2} \times$ *their* $7.37 \times \sqrt{9^2 - (\text{their } 7.37)^2}$
13(b)(iv)	19.6 to 19.7 **nfww** 	3	**M1** for $\tan 35 = \frac{PR}{\text{their } RQ}$ oe or $\frac{PR}{\text{their } RQ} = \frac{\text{their } RB}{\text{their } AR}$ oe where *their RQ* = (*their* 8.72 – *their* 7.37) **M1** for *their* area ARB + $\frac{1}{2} \times$ *their RQ* $\times$ *their PR*

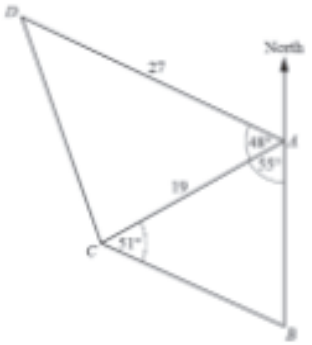
14(a)	Need to see 2.58 rounded from a correctly obtained 2 581 or better.	3	Method 1 **M2** for $AY = 3.65\cos 45$ or $(3.65 \div 2) \div \sin 45$ **or** **M1** for e.g. $\frac{AY}{3.65} = \cos 45$ or $sin45 = \frac{3.65 \div 2}{AY}$ Method 2 **M1** for such as $AY^2 + AY^2 = 3.65^2$ or $3.65^2 + 3.65^2 = AC^2$ soi **M1** for $AY^2 = \frac{3.65^2}{2}$ oe **A1** for $AY = 2.580[9\ldots]$
14(b)	7.93	2	**M1** for $7.5^2 + 2.58^2$
14(c)	26.6° or $2\sin^{-1}\left(\frac{0.5\times 3.65}{their 7.93}\right)$	3FT	**M2** for $2\sin^{-1}\left(\frac{0.5\times 3.65}{their 7.93}\right)$ or $\cos[\ldots] = \frac{their 7.93^2 + their 7.93^2 - 3.65^2}{2\times their 7.93^2}$ **Or** **M1** for $\sin[\ldots] = \frac{0.5\times 3.65}{their 7.93}$ or $3.65^2 = \text{their}7.93^2 + \text{their}7.93^2 - 2\times\text{their } 7.93^2 \times \cos[\ldots]$
14(d)(i)	11.18 or 11.2	2	**M1** for $\tan 77 = \frac{XY}{2.58}$ oe
14(d)(ii)	80.7°	2FT	**M1** for $\tan[\ldots] = \frac{their\ 11.2}{3.65 \div 2}$
15(a)	137	1	
15(b)	085	1	

16(a)	7040 or 7035. …	3	**M1** for $\frac{1}{2}\times100\times70$ oe **M1** for $\frac{1}{2}\times100\times110\times\sin40$ oe
16(b)	374 or 375 or 374.4 to 374.5….	5	**M2** for $110^2+100^2-2\times110\times100\times\cos40$ oe or **M1** for implicit form **A1** for 5250 or 5247. … (or 72.4 or 72.43 to 72.44) **M1** for 70^2+100^2
16(c)	64.3 or 64.27 to 64.28 nfww	2	**M1** for $\sin40=\frac{\text{distance}}{100}$ oe
16(d)	235	3	**B2** for [angle ACB =] 34.99 to 35 or [angle ABC =] 55[.0…] or **M1** for $\tan[ACB]=\frac{70}{100}$ or $\tan[ABC]=\frac{100}{70}$ or equivalent trig ratio

17(a)(i)	14.4[2…]	2	**M1** for 12^2+8^2
17(a)(ii)	128.6° to 129°	3	**M1** for $\tan\theta=\frac{12}{15}$ or $\tan\theta=\frac{15}{12}$ **A1** for 38.6 to 38.7 or 51.3 to 51.4 After A0, **SC1** for $90+\tan^{-1}(\frac{12}{15})$ evaluate or $180-\tan^{-1}(\frac{15}{12})$ evaluated
17(b)(i)	472 to 488	2	**B1** for 6.3 to 6.5 seen
17(b)(ii)	F correctly placed	2	**M1** for either TF = 6 cm plotted or correct angle
17(b)(iii)	242° to 248°	1	

18(a)	140°	2	**M1** for 180 – (360 ÷ 9) or 180(9 – 2) ÷ 9
18(b)(i)	21.89.... with at least $7^2+18^2-2\times7\times18\times\cos115$ seen	3	**M1** for $7^2+18^2-2\times7\times18\times\cos115$ **A1** for 479.5 or 373 + 106.49.. or 373 + 106.5
18(b)(ii)	18.8° to 19°	3	**M2** for $\sin B=\frac{11\sin28}{16}$ or **M1** for $\frac{\sin B}{11}=\frac{\sin28}{16}$ oe
18(b)(iii)	95.47° to 95.5°	4	**B3** for 84.5 to 84.6 or **M2** for $\sin E=\frac{109\times2}{\text{their } DE\times21.9}$ or **M1** for $109=\frac{1}{2}\times21.9\times\text{their } DE\times\sin E$

19(a)	14.96 to 15[.0] nfww	3	**M2** for $15.1^2 - 2^2$ (= 224.01) or **M1** for $DC^2 + 2^2 = 15.1^2$ or 15.1^2 – *their* 2^2 with horizontal line seen or **B1** for horizontal line and 2 soi
19(b)	97.46 to 97.55	3	**M2** for $\cos[A] = \frac{9^2 + 11^2 - 15.1^2}{2 \times 9 \times 11}$ oe or **B1** for $15.1^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos[A]$ oe
19(c)	123.8 to 124.1 nfww	4	**M3** for $\frac{1}{2} \times 9 \times 11 \times \sin(b) + \frac{1}{2} \times (4+6) \times (a)$ oe with (a) ≠15.1 soi or **M1** for $\frac{1}{2} \times 9 \times 11 \times \sin$ **(b)** oe soi and **M1** for $\frac{1}{2} \times (4+6) \times (a)$ oe with **(a)** ≠ 15.1 soi
19(d)	495.5 to 497	2	FT **(c)** × 4 **B1** for (figs 5)^2 soi

20	(a)	$[DT =]10.8$ or 10.816 to 10.82	2	**M1** for $DT^2 = 6^2 + 9^2$ oe
	(b)	139 or 139.2 to 139.3	3	**B1** for $BT = 10$ **M1** for sum of areas of four triangles seen, with at least 3 of the following correct: $\frac{1}{2}\times 8\times 6$, $\frac{1}{2}\times 9\times 6$, $\frac{1}{2}\times 8\times \textit{their } DT$, $\frac{1}{2}\times 9\times \textit{their } BT$
	(c)	504	2	**M1** for $9\times 8\times 5$ or $\frac{1}{3}\times 9\times 8\times 6$
	(d)	50.7° final answer	3	**M1** for finding an acute angle in triangle THG. e.g. $\tan[\ldots] = \frac{11}{9}$ or $\tan[\ldots] = \frac{9}{11}$ **A1** for 50.7[...]° or 39.28 to 39.3°
21	(a)	283°	1	
	(b)	055°	1	
	(c)	$[AB =]$ 15.4 or 15.36[...]	3	**B1** for $ABC = 74°$ **M1** for $\frac{AB}{\sin 51} = \frac{19}{\sin ABC}$
	(d)	$[DC =]$ 20.08 to 20.1	3	**M2** for $[DC^2 =] 19^2 + 27^2 - 2\times 19\times 27\times \cos 48$ or **M1** for cosine formula with one error
	(e)	Correct working leading to 114 minutes or 1 hour 54 minutes 	4	**M1** for $AX = 19\times\cos 48$ or for $CX = 19\times\sin 48$ **M1** for $DX = 27 - \textit{their } AX$ Or for $DX = \sqrt{\textit{their } DC^2 - \textit{their } CX^2}$ **M1** for Time $= 216\times\frac{\textit{their } DX}{27}$ oe

22 (a) (i)	2.67	**2**	M1 $\frac{AD}{3} = \cos 27$ oe
(ii)	4.57	**3**	M2 for $CD = \frac{3}{\sin 41}$ oe or M1 for $\frac{3}{CD} = \sin 41$ oe
(b)	53.1 126.9	**3**	M1 for $\frac{1}{2} \times 3 \times 5 \times \sin P\hat{Q}R = 6$ oe and A1 for 53.1 or SC1 for supplementary angles from sin $P\hat{Q}R = k$.
23 (a)	326 ft	**4ft**	M2 for $65^2 = 110^2 + 70^2 - 2 \times 110 \times 70 \times \cos A\hat{C}B$ soi or M1 for the cosine rule with one error. and A1 for 33.9 or 146.1 or 59.2 and B1 ft for 360 – their $A\hat{C}B$ oe SC 2 for 109.1 or 37.0
(b)	92.2	**3**	M2 for $\frac{AD}{\sin(70+58) or (180-(70+58))} = \frac{110}{\sin 70}$ oe soi or M1 for 70 + 58 or 180 – (70 + 58)
(c) (i)	13.6 or 13.7	**2**	M1 for tan $YBC = \frac{17}{70}$ or tan $BYC = \frac{70}{17}$
(ii)	16.5	**3**	M1 for Figs $\frac{110}{24}$ soi and B1 for × by $\frac{60 \times 60}{1000}$ oe soi

24	280, 295, 310	3*	**C2** for two correct values OR **B2** for two from 70°, 40° and 55° seen OR **B1** for 70° seen or for 10° or 120° correctly positioned on diagram

25 **(a)**	15.1 or 15.08(…..	**3**	M1 for $\tan\theta = \frac{31}{115}$ or $\tan\theta = \frac{115}{31}$ A1for $\theta = 15.1$ or $\theta = 74.9$
(b) (i)	18.8 or 18.77……	**2**	M1 for $\sin\theta = \frac{354}{1100}$
(ii)	251 or 251.2(…….	**1ft**	270 – their $L\hat{J}K$ final ans.

26 **(a) (i)**	5.38 to 5.39 or √29	**2**	M1 for $(AC^2) = 2^2 + 5^2$
(ii)	0.517 to 0.518	**2**	M1 for $\frac{CE}{2} = \sin 15$ oe
(iii)	68.8 to 68.9	**4**	M1 for $\frac{AF}{2} = \cos 15$ oe or $BC^2 = BE^2 +$ (their CE)² any complete alternative method A1 for 1.932 and M1 for $\tan F\hat{A}E = \frac{5}{2\cos 15}$ oe or $\frac{5}{their(AF}$
(b) (i)	80.9(4…. Or 81	**3**	B1 for $10^2 = 6^2 + 9^2 - 2\times6\times9\times\cos\theta$ or B2 for $\cos\theta = \frac{9^2 + 6^2 - 10^2}{2\times9\times6}$
(ii)	>	**1**	